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Warranty cost estimation of a multi-module product

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Abstract *This article deals with the problem of cost estimation for increased warranty time of a multi-module product. The warranty policy of interest is two-dimensional involving warranty limits on both age and usage of the product. Failure of the product is caused due to malfunctioning of its module(s). Warranty service is rendered through repair or replacement of the respective module(s). From the past data, it is observed that age and usage are highly correlated. Based on life (age) data, the joint life distribution of the modules is well described by multivariate exponential distribution of Marshall and Olkin. The same is utilized to estimate cost for desired warranty times by the method of simulation.*

Introduction

The warranty concept is important to both the manufacturer and the customer of virtually any consumer or commercial product. Product warranty provides protection to customer as recourse for dealing with items that fail to fulfill their intended purpose, usually in the form of some stated compensation offered by the manufacturer. The basic intention is to protect the customer from shoddy or unreliable goods. On the other hand, warranty is treated by the manufacturer as a marketing strategy that creates better customer satisfaction, which finally helps to get hold of a bigger market share. In general, warranty is viewed as a contractual agreement between manufacturer and customer in connection with the sale of a product.

Consequently, management of warranty has drawn the attention of industrial houses and researchers alike, the prime objective being estimation of warranty cost (or the cost to service warranty). The basic input for the same are: life distribution (and hence reliability function) of the concerned product; and warranty policy of interest. Warranty policy is a statement about the kind and extent of compensation that would be provided by the manufacturer to its customer. It can be either simple or complex depending upon the type of

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product under consideration. However, all warranty policies are governed by the following two basic features:

(1) *Warranty limit*. It specifies the length of time period, since the sale of the product, during which warranty is provided by the manufacturer. A warranty policy may be one-dimensional or two-dimensional. A one-dimensional (1D) policy is always based on just one of the variables – age or usage of the product. In contrast, a typical two-dimensional (2D) policy is stated using both age as well as usage. Whatever may be the case, a policy statement explicitly specifies the warranty limit(s) of the corresponding variable(s). It may be noticed that a 1D policy is a special case of a 2D policy with unlimited warranty time for one of the variables.

(2) *Compensation scheme*. The kind of compensation to the customer (in the event of non-performance of the product within warranty limit) can be in the form of repair and/or replacement free of cost. Also, it can be on pro-rata basis.

A taxonomy to classify different warranty policies is available in Blischke and Murthy (1994).

There is a very extensive literature dealing with the subject – product warranty. A variety of warranty policies and the mathematical models for analyzing various related engineering and management issues are discussed in Blischke and Murthy (1994). They also provide a framework for the entire warranty program. Blischke and Murthy (1996) give a comprehensive treatise of consumer product warranties viewed from different perspective. On the whole, 1D policy has received a lot of attention, and many different aspects have been studied by researchers from different disciplines. A review of the same can be found in Murthy and Blischke (1992). For mathematical analysis of 2D policies, we refer to Blischke and Murthy (1994), Murthy et al. (1995) etc. Related issues concerning warranty, like warranty reserve, warranty servicing, marketing aspect etc. are also studied by many researchers. For example, Menezes and Currim (1992) treat warranty as a marketing variable and focus on matters that a manager should take into account in order to determine the length of warranty; Ja et al. (2002) discuss management of warranty reserve under different compensation schemes for the customer; preventive maintenance and replacement strategies are studied by Sahin and Polatoglu (1998).

Almost the whole of the existing literature has focused on the warranty analysis of single component or item consisting of single module. But in practice, there are many items that are being covered by warranty, have more than one component or module. Today, nearly all complex items and systems are built with modular structure, a module being a collection of components.

Blischke and Murthy (1994, p. 73) provide a brief discussion on the problems pertaining to this case.

In this article, we deal with a real-life situation where estimation of warranty cost for a multi-module product is under consideration. We first state the exact problem of warranty cost estimation. Then, the assumptions are enumerated in order to describe the set-up. Subsequently, we illustrate the methodology to estimate warranty cost. Finally, the results are presented.

Problem description

The product of interest is starter motor manufactured by Lucas TVS Ltd, India, for a particular application in vehicle. The only function of starter motor is to draw power from battery and start the engine. Subsequently, it remains off-life. The product consists of seven modules, e.g. solenoid, field coil, armature, etc. Each module is essentially an assembly. We denote these modules by A_1, A_2, . . ., A_7.

Presently, this product is covered by 2D repair warranty policy, where the dimensions are given by age (W) and usage (U) with respective limits as 365 days and 1,000 hours. The warranty policy reads like this – free service will be provided to the customer in the event of any functional failure of starter motor for a period of 365 days or 1,000 hours (of running the engine) whichever is earlier, from the time of commissioning. Following the terminology of Murthy et al. (1995), this is referred to as "closed" 2D policy.

The company is contemplating to increase the existing warranty limits, in order to provide superior customer service, and meet the ever-increasing market competition ahead. The immediate implication of increasing the limits is an additional financial burden to the company. If this additional cost is very high, it is essential that design (of the product) be improved to reduce the incidence of failure. Alternatively, the company can go ahead with the present design. Therefore, the problem under consideration is to estimate the warranty cost (per unit of the product) for an extended warranty period. We denote this cost by WC. Observe that WC is a random variable that depends on the number of failures during the proposed warranty period. The expected value of WC, $E[WC]$ is important in decision-making purpose, like pricing of the product, planning for warranty reserve etc. Consequently, we intend to determine $E[WC]$, or more precisely, $E[WC(W_0,U_0)]$ where W_0 = warranty limit on age, U_0 = warranty limit on usage, and $WC(W_0,U_0)$ is the warranty cost for the limits W_0 and U_0. Under the existing warranty, $W_0 = 365$ days and $U_0 = 1{,}000$ hours. The value of $E[WC(W_0,U_0)]$ is of interest when $(W_0, U_0) = (365, \infty)$, (730, 2,000) and (730, ∞). Notice that $U_0 = \infty$ implies unlimited use, and therefore, the policy becomes one-dimensional.

Definitions

In the following we define some terminology and concept that will be referred to throughout this article:

- *Life*. The life of the product, in strict sense, is the number of times it has been used to start the engine. However, from customers' point of view, the trouble-free period (measured by age or usage) describes its life. Therefore, life is represented by age as well as usage.
- *Failure mode*. Any failure of the product is caused by the failure of a single module or a combination of the modules. This cause is termed as failure mode.
- *Repair*. The rectification of a failed starter motor involves either minimal repair or replacement of the respective module(s). Subsequently, the product is the same as it was before the failure.
- *Downtime*. The time period elapsed between the occurance of failure and the completion of repair is called downtime. The downtime includes delay in reporting, failure diagnosis, repair time and administrative delay. During this period, the product is not exposed to risk of failure.

The set-up

It may be recalled that the product under consideration is presently covered by warranty. This has generated a large volume of information in the form of warranty claim data. We analyze them in order to extract different aspects of failure mode, repair cost, downtime, etc. that define the problem environment.

Basic data and summarization

The company maintains the complete database with regard to every claim of warranty cost arising out of free repair/replacement service. A typical record of warranty claim contains the information:

- job number;
- chasis number (of the vehicle);
- month/year of manufacturing (of the starter motor);
- date of commissioning;
- date of failure;
- date of job completion;
- failure mode;
- hours covered (usage);
- material cost;
- labour cost;
- handling charges; and
- total repair cost (= material cost + labour cost + handling charges).

All the claim details and monthly production figures are available for a period of consecutive 54 months, say, $M_1, M_2, \ldots, M_{54}$ during the recent past. These constitute the "basic data":

- *Selection of study period*. It is to be noted that a starter motor takes about 15 months from its production month to cover the warranty phase – about three months on transit before commissioning, and then 12 months of warranty period. Therefore, any starter motor produced in M_{40} or beyond may still be requiring at least another month to yield complete information on its failure during its warranty period. Because of such incompleteness in information, we ignore the data set beyond M_{39}.

  Now, we consider the database for the period M_1 to M_{39}. We try to identify (if any) major change in failure pattern over time. For this purpose, we define, for any month (say j):

  $$\text{Field-Return of } M_j = \frac{\#\text{ of claims w.r.t. all starter motors produced during } M_j}{\text{Total production during } M_j}.$$

  The trend in monthly field-return for the period M_1 to M_{39} is given in Appendix 1. It shows a significant drop in M_{25}, but remains more or less stable thereafter. This change is directly attributed to the *design modification* carried out by the Engineering Department of the company.

  Consequently, we decide to take the production period M_{25} to M_{39} as the "study period" that provides the relevant information. During this period, the total production is 34,348 and the corresponding number of claims is 247. All subsequent discussion in this section is based on this information during the study period.

- *Price adjustment*. We take note of increase in material cost and labour charge from time to time. We have gathered all the relevant figures and used them to bring (adjust) every single warranty cost figure to the current price level of M_{39}.

Thus, on summarization, every claim provides the information:

- claim ID;
- age (in days);
- usage (in hours);
- failure mode;
- downtime (in days); and
- (adjusted) repair cost.

On scrutiny of all the claims, it is observed that "no starter motor required warranty service more than once". Further, the value of $E[WC(365, 1{,}000)]$ is estimated as Rs. 2.29.

Analysis and observations

The above claim data correspond to the warranty limits of $W_0 = 365$ days and U_0 =1,000 hours, and therefore, information on the product life is time-censored. All the following observations hold good for the existing warranty period:

- *Relationship between age (W) and usage (U).* Obviously, U is a function of W satisfying $U = 0$ when $W = 0$, i.e. with zero-intercept. On investigation, it is found that the model $U = \alpha W^{\beta}$ describes the relationship between age and usage extremely well (refer to Appendix 2 for ANOVA).The estimates[1] of the parameters are $\hat{\alpha} = 3.3774$, $\hat{\beta} = 0.9728$. Evidently, the above relationship can be used to predict usage with quite high degree of accuracy.
- *Failure mode (s) and repair cost (C_s).* Of the seven modules, A_7 has not failed at all. Table I gives the failure modes with the corresponding observed frequencies of failure. A_8 represents the failure mode of the starter motor requiring "general service". For every failure mode s, the empirical distribution of repair cost (C_s) is obtained. They are found to be distinct.
- *Association of repair cost (C_s) and age (W).* For every given failure mode s, the association between C_s and W has been studied using χ^2-test. We observe that they have no significant association.
- *Downtime distribution.* The empirical distribution of downtime (D) is extracted. It is found to vary up to 240 days, the average being 37 days.
- *Association of downtime (D) with failure mode (s).* It may be expected that depending upon the failure mode, the distribution of downtime may vary.

Failure mode Code (s)	Name	No. of failures (n_s)
1000000	A_1	11
0100000	A_2	14
0010000	A_3	27
0001000	A_4	44
0000100	A_5	16
0000010	A_6	15
0000001	A_8	64
1001000	A_1A_4	3
1000001	A_1A_8	6
0101000	A_2A_4	2
0100001	A_2A_8	4
0010010	A_3A_6	13
0010001	A_3A_8	28
Total cases (n_0)		247

Note: Total production (n) = 34, 348

Table I.
Failure modes

However, we notice that downtime pattern is same over all the failure modes.

- *Association of age (W) and downtime (D).* For any given failure mode, we also look for possible association between age and downtime. We find them to be independent.

Consequently, we have identified the failure modes, and the empirical distribution of repair cost corresponding to every failure mode, the empirical distribution of downtime, etc. Also, we have observed that age and usage are highly correlated. We define the set-up in terms of the above observations, that is, "we assume that they hold good for the extended warranty period also".

The method

The present problem of warranty cost estimation pertains to closed 2D repair policy for a multi-module product. The basic modelling approach to 2D policy (also to 1D policy) has been based on stochastic process. Specifically, it involves the determination of number of renewals (failures) on the warranty region given by $(0, W_0) \times (0, U_0)$ where W_0 and U_0 are the warranty limits of interest. Blischke and Murthy (1994) have presented a detail analysis of the same for single-module product with zero or negligible downtime. The two basic steps involved are: identification of product life distribution; and computation of expected number of renewals during the warranty period. Analytical solution for the general case looks extremely difficult, and therefore, numerical procedure (for the second step) is recommended. Our problem is further complicated, because of several reasons, e.g.: it is concerned with multi-module product; repair cost is variable, and dependent on failure mode; downtime is significantly large; and so on.

Consequently, our approach to the problem is to identify the "joint life distribution" of the modules, and then use it to estimate the warranty cost for the extended period of interest by method of simulation. It is important to note that joint life distribution will be described by age only, since it is highly correlated with usage. However, (predicted) usage value will be contrasted with its limit during simulation in order to assess the validity of warranty claim.

Joint life distribution of modules

Since we have time-censored life data, we confine ourselves to the class of parametric models only. Identification of a model involves:

- selection of a standard model by making use of the characteristic of the present situation;
- estimation of the associated parameters; and
- validation of the model:

Model selection. We know that starter motor (henceforth call it "system") consists of several modules. The functional failure of the system is caused by failure of either a module or a combination of modules. This feature of joint failure of modules is very important. The multivariate exponential (MVE) distribution of Marshall and Olkin (1967) possesses this property. Therefore, we intend to assess the suitability of the same for our data. In the following, we describe this distribution.

Let k be the number of modules. Denote X_i as the life (age) of module i, $i = 1, \ldots, k$. Then:

$$P[X_1 > x_1, \ldots, X_k > x_k] = \exp\Big[-\textstyle\sum_1^k \lambda_i x_i - \sum_{i<j} \lambda_{ij} \max\{x_i, x_j\} - \sum_{i<j<l} \lambda_{ijl} \max\{x_i, x_j, x_l\} \; \vdots \; - \lambda_{12\ldots k} \max\{x_1, x_2, \ldots x_k\}\Big] \tag{1}$$

gives the MVE distribution of Marshall and Olkin. For $k = 2$:

$$P[X_1] > x_1, X_2] = \exp[-\lambda_1 x_1 - \lambda_2 x_2 - \lambda_{12} \max\{x_1, x_2\}] \tag{2}$$

where λ_1 (λ_2) is rate of failure for module 1 (equation (2)), and λ_{12} is the rate of joint failure of modules 1 and 2. When $k > 2$, the values for λ_s have similar interpretation.

A compact form of equation (1) is given by:

$$P[X_1 > x_1, \ldots, X_k x_k] = \exp\Big[-\textstyle\sum_{s \in S} \lambda_s \max(s_i x_i)\Big] \tag{3}$$

where S is the set of vectors $(s_1, \ldots, s_k)$ with each $s_i = 0$ or 1 but $(s_1, \ldots, s_k) \neq (0, \ldots, 0)$.

Estimation of model parameters. It must be noted that the probability distribution given by equation (1) or (3) is not absolutely continuous for $k \geq 2$. Hence, it poses some difficulty in estimation of the parameters involved. Many researchers have studied the bivariate case ($k = 2$) of the problem, but the general case (arbitrary k) is considered by Arnold (1968) and Proschan and Sullo (1976). Both these articles assume uncensored life data, i.e. every system is observed to fail. And hence, they do not apply here. In the following, we present a procedure analogous to Arnold's (1968) approach to estimate the parameters under censoring.

Note that W represents the life of the system. Therefore, $W = \min\{X_1, \ldots, X_k\} \cap \text{Exp}(\lambda)$ where $\lambda = \sum_{s \in S} \lambda_s$. Let W_0 be the censoring time. Then, we observe the following:

(1) $P[W \leq W_0] = 1 - \exp(-\lambda W_0)$.

(2a) For fixed i:

$$P[X_i < X_j \forall j \neq i; W \leq W_0] = \frac{\lambda_s}{\lambda}[1 - \exp(-\lambda W_0)],$$

where s is such that $s_i = 1$ and $s_j = 0 \forall j \neq i$.

(2b) For fixed i and j:

$$P[X_i = X_j <; X_k \forall k \neq i,j; W \leq W_0] = \frac{\lambda_s}{\lambda}[1 - \exp(-\lambda W_0)],$$

where s is such that $s_i = s_j = 1$ and $s_k = 0 \forall k \neq i,j$.

And so on.

For large sample size, we equate the above population quantities to the respective sample quantities to estimate the parameters, i.e.:

$$1 - \exp(-\lambda W_0) = \frac{n_0}{n}, \text{and} \quad (4)$$

$$\frac{\lambda_s}{\lambda}[1 - \exp(-\lambda W_0)] = \frac{n_s}{n}, \text{for all } s \in S. \quad (5)$$

where n = sample size, n_0 = total number of failures by time W_0, and n_s = number of failures by time W_0 on failure mode s. Of the seven modules in the starter motor, A_7 has not failed at all (see Table I). Without loss of generality, we consider that the system consists of six modules. Besides, we assume that any failure of the system requiring "general service" is the failure of a hypothetical module, so that, we take $k = 6 + 1 = 7$.

We have $W_0 = 365$ days and the values of n, n_0 and n_s are available in Table I. With these data, we solve the equations (4) and (5) for values of λ and λ_s. The estimates are given in Table II.

Model diagnostics. We now present checks for adequacy of the MVE distribution to know whether it represents our data. This is done in the following two steps:

(1) We carry out the usual goodness-of-fit test for system life. According to the model, it follows exponential distribution with parameter λ. Appendix 3 gives all the relevant calculations, and we observe that there is no evidence to believe otherwise.

(2) We compare the model and our data using the criterion: "conditional probability of occurrence of s-th failure mode given that the system has failed". That is, for every time-interval (t_1,t_2) (say, Δt), test:

$$H_0 : p(\Delta t) = p_0(\Delta t) \text{ vs. } H_1 : p(\Delta t) \neq p_0(\Delta t),$$

Failure mode Code (s)	Name	No. of failures (n_s)	Failure rate (λ_s/day)
1000000	A_1	11	0.8806E-06
0100000	A_2	14	1.1207E-06
0010000	A_3	27	2.1614E-06
0001000	A_4	44	3.5223E-06
0000100	A_5	16	1.2808E-06
0000010	A_6	15	1.2008E-06
0000001	A_8	64	5.1233E-06
1001000	A_1A_4	3	0.2402E-06
1000001	A_1A_8	6	0.4803E-06
0101000	A_2A_4	2	0.1601E-06
0100001	A_2A_8	4	0.3202E-06
0010010	A_3A_6	13	1.0407E-06
0010001	A_3A_8	28	2.2414E-06
System (n_0)		247	19.7728E-06

Note: The remaining values for λ_s are at zero level

Table II. Estimated failure rates

where $p_0(\Delta t)$ = conditional probability of occurrence of s-th failure mode during Δt given that the system has failed

$$= \frac{\lambda_s}{\lambda},$$

and $p(\Delta t)$ = the corresponding observed value of $p_0(\Delta t)$

$$= \frac{\text{\# of cases of failure mode } s \text{ during } \Delta t}{\text{\# of failures of the system during } \Delta t}$$

$$= \frac{n_s(\Delta t)}{n(\Delta t)}, \text{say.}$$

This is a binomial-based test. We take its normal approximation for large $n(\Delta t)$. The test statistic:

$$Z(\Delta t) = \frac{n_s(\Delta t) - n(\Delta t)p_0(\Delta t)}{\sqrt{n(\Delta t)p_0(\Delta t)(1 - p_0(\Delta t))}},$$

follows standard normal distribution. This test is carried out for every failure mode. We note that we cannot distinguish between the model and our data.

Simulation
Having identified the joint life distribution of the modules, we proceed to carry out simulation by making use of the observations (a)-(f). The input required by the simulation procedure are: warranty limits of interest; joint life (age) distribution of the modules; relationship between age and usage; distribution of repair cost for every failure mode; and downtime distribution. And, output is warranty cost (WC) for one unit of the product. This procedure is repeated for a specified number of units, and the average of values for WC is taken as the estimate of $E(WC)$.

The procedure is presented below in the form of an algorithm. The corresponding flow diagram is in Appendix 4. In brief, the procedure is as follows. Given a new unit, its first failure time is obtained by simulation of the joint life distribution of modules. Note that the first failure time is same as the age of the unit. The usage of the unit is then predicted corresponding to this age. If either age or usage exceeds its corresponding warranty limit, warranty service is no more applicable to the unit, and the procedure stops. Else, warranty service is undertaken. Consequently, a repair cost is recorded (by simulating the distribution depending upon the failure mode); simulated downtime is added to the age at failure to obtain the age on completion of repair. Likewise, subsequent failures are simulated, age of the unit is updated and repair cost is recorded as long as warranty service is applicable to the unit. As and when either age or usage exceeds its warranty limit, the procedure stops, and the accumulated cost of repair(s) is given as the value of WC.

Algorithm for simulation

Step 1. Let W_0 and U_0 be the warranty limits of interest for age and usage respectively. Set current time $CT \leftarrow 0$ and warranty cost $WC \leftarrow 0$.

Step 2. Simulate the MVE distribution. Let W be the failure time of the system, and s be the failure mode. Update $CT \leftarrow CT + W$.

Step 3. If $CT > W_0$ (i.e. failure beyond warranty limit of age), go to step 8.

Step 4. Corresponding to CT, predict U (usage). [Observation (a) is used.]

Step 5. If $U > U_0$ (i.e. failure beyond warranty limit of usage), go to step 8.

Step 6. Simulate the repair cost distribution of failure mode s. Let C_s be the observation. Update $WC \leftarrow WC + C_s$. [Observations (b) and (c) are used.]

Step 7. Simulate downtime distribution. Let D be the observation. Update $CT \leftarrow CT + D$. Go to step 2. [Observations (d), (e) and (f) are used.]

Step 8. Return WC as the warranty cost for the limits W_0 and U_0.

Simulation of MVE distribution in step 2 is carried out with the help of the "representation theorem" (Marshall et al., 1967). In step 4, the predicted value is the sum of the explained part (computed directly from the regression equation) and the unexplained part (obtained by simulating the error distribution corresponding to the regression).

Further, it is to be noted that the algorithm does not impose any restriction on the frequency of failures of any unit. Whereas, not a single starter motor has failed more than once within the existing warranty time.

Computer coding of the algorithm is done in Turbo Pascal. Its built-in random number generator is used for simulation of random variates from the respective distributions. The performance of this generator with regard to the present model is observed to be quite satisfactory.

Results and discussion

We use the above simulation model in order to obtain estimate of E[$WC(W_0,U_0)$] for $(W_0, U_0) = (365, \infty)$, (730, 2000) and (730, ∞). Besides estimating E[$WC(W_0,U_0)$], we derive the standard error (s.e.) of the estimate for every (W_0,U_0) as follows. Compute an estimate of E[$WC(W_0,U_0)$] based on 30,000 starter motors (approximate quantity of annual production). Likewise we obtain 200 estimates. The grand average of these values is taken as the estimate of E[$WC(W_0,U_0)$], and standard deviation of the values gives s.e. of the estimate. The results are summarized in Table III for existing as well as the new warranty limits of interest.

This has been found to be quite vital input to the management. However, it is important to note that with the increase in warranty time, new failure modes may appear resulting in an additional cost. It is suggested that the management should explore the possibility of obtaining an estimate of the same to take a final decision on the revision of warranty time or otherwise. Besides, the management take due note of the failure rates of respective modules for initiating improvement action in order to reduce the incidence of failure.

Conclusion

In this article, we have studied the problem of warranty cost estimation for a multi-module product in real-life situation. The joint life distribution of modules is modelled by MVE distribution of Marshall and Olkin (1967).

	Warranty limits (W_0,U_0)			
	Existing	New		
Estimates	(365, 1000)	(365, ∞)	(730, 2,000)	(730, ∞)
Average	2.15	2.42	4.25	4.82
s.e. of average	0.16	0.17	0.18	0.26
95% confidence interval	(1.84-2.46)	(2.09-2.75)	(3.90-4.60)	(4.31-5.33)

Table III. Estimates of E[$WC(W_0,U_0)$] (in Rs.)

Subsequently, this distribution is used in simulation to estimate warranty cost for the desired extended period.

Implicitly, we have presented a general approach for warranty cost estimation of the two-dimentional repair warranty policy of a multi-module product. It can be seen that the joint distribution of the modules, or even the joint distribution between age and usage can be arbitrary. Further, the presence of dependency among age, downtime, failure mode etc., only calls for the respective conditional distributions.

Note

1. Since ρ^2 is very high, it is expected that there will be numerical problem in obtaining maximum likelihood estimates. Consequently, least square estimates are computed assuming the absence of censoring.

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Appendix 1

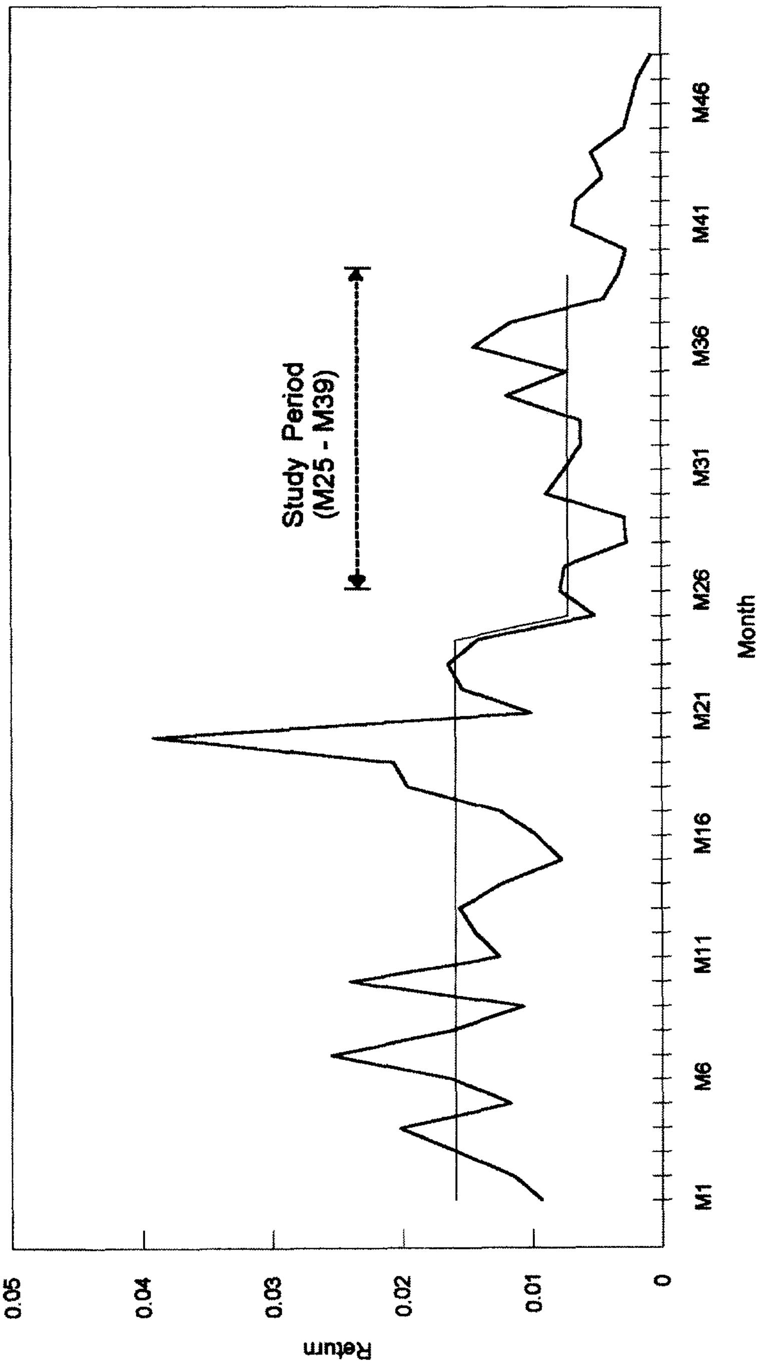

Figure A1.
Monthly field return trend

Appendix 2

Table AI. Regression analysis (Multiplicative model: $U = \alpha W^{\beta}$)

Dependent variable (U)	:	Usage (in hours)
Independent variable (W)	:	Age (in days)
$\hat{\alpha}$	:	3.3774
$\hat{\beta}$	:	0.9728
$\hat{\rho}$	:	0.9709
$\hat{\rho}^2$	:	0.9427

Note: $\rho =$ corr. coeff. between $\ln W$ and $\ln U$

Table AII. Regression analysis (analysis of variance table)

Source	Deg. of freedom	Sum of squares	Mean square	F-ratio
Model	1	358.3516	358.3516	4030.95[a]
Error	245	21.7876	0.0889	
Total	246	380.1392		

Note: [a] The critical value for 1 per cent level of significance is 6.69

Appendix 3

Table AIII. Goodness-of-fit for system life (age)

Age (days)	Observed frequency	Expected frequency	Computed χ^2-value
000-030	23	20.37	0.34
030-060	16	20.36	0.93
060-090	17	20.34	0.55
090-120	15	20.33	1.40
120-150	24	20.32	0.67
150-180	24	20.31	0.67
180-210	21	20.30	0.02
210-240	22	20.28	0.15
240-270	21	20.27	0.03
270-300	26	20.26	1.63
300-330	16	20.25	0.89
330-365	22	23.61	0.11
Above 365	34,101	34,101	0.00
Total	34,348	34,348	7.39[a]

Note: [a] The critical value for 5 per cent level of significance is 19.68

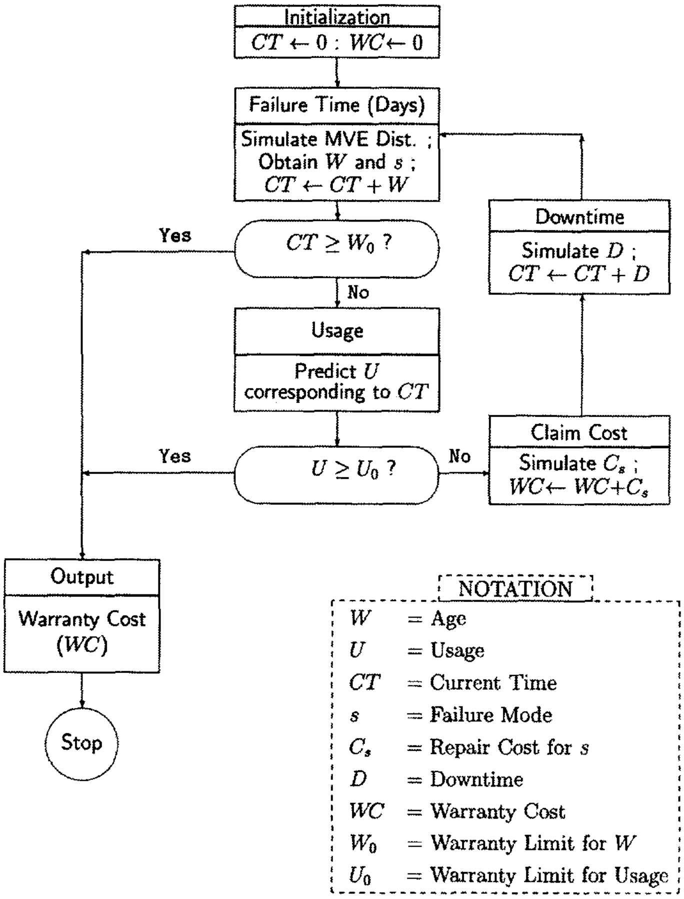

Figure A2. Flow diagram for simulation